Comments on "Energy and Power Orthogonality in Isotropic, Discretely Inhomogeneous Waveguides"

Paul R. McIsaac, *Member, IEEE*

In a recent letter,[1] Manring and Asmussen discuss orthogonality relations for modes in cylindrical waveguides containing isotropic media whose permittivity may be discretely inhomogeneous. They develop these relations in terms of the TE and TM portions of the transverse electric and magnetic fields. As an example, they discuss the TE and TM modes of certain inhomogeneously-filled waveguides (for example, the axisymmetric modes in a coaxially-loaded circular cylindrical waveguide). The authors show that for the transverse electric fields of the TM modes:

$$\int\int \hat{\varepsilon} E_{t_i}^{\mathrm{TM}} \cdot E_{t_j}^{\mathrm{TM}} ds = 0, \qquad i \neq j, \tag{1a}$$

while for the electric fields of the TE modes:

$$\int\int \frac{1}{\hat{\mu}} E_{t_i}^{\mathrm{TE}} \cdot E_{t_j}^{\mathrm{TE}} ds = 0, \qquad i \neq j. \tag{1b}$$

These are equations (14) and (15) of [1]; analogous equations for the transverse magnetic fields are also given in [1].

The authors then state: "On the basis of these equations, the cavity orthogonality equations given by Harrington are not valid in general [1, p. 432]. The equation given for the electric field is valid only for TM modes while the equation given for the magnetic field is valid only for TE modes." The cavity orthogonality relations referred to are equations (8-163) and (8-164) in [1]. These state that the modes for a closed cavity of arbitrary shape with discretely inhomogeneous media satisfy the relations:

$$\int\int\int \varepsilon E_i \cdot E_j^* d\tau = 0, \quad \int\int\int \mu H_i \cdot H_j^* d\tau = 0, \quad i \neq j. \tag{2}$$

These latter equations are general and are valid, in particular, for the example introduced by the authors. The apparent disagreement between (1b) and (2) for the electric fields for TE modes is not a real disagreement. Each of these equations, properly applied, is valid. The orthogonality relations of (1) are for waveguides; these hold for two waveguide modes at the same frequency which have different axial phase constants ($k_{zi} \neq k_{zj}$). The cavity mode orthogonality relations of (2) hold for the fields of two distinct cavity modes with (usually) different resonant frequencies.

For the type of cavity under discussion (a section of waveguide with end walls, Fig. 1) each cavity mode will have an integer number of half wavelengths between the end walls. The volume integrals in (2) include a radial integration, an azimuthal integration (which is trivial since the fields are axisymmetric) and an axial integration. In general, the resonant frequencies of the two modes will differ. If the two modes have different numbers of half wavelengths between the end walls, then the axial integration will yield zero (regardless of the radial variation of the fields). If the two modes have the same number of half wavelengths between the end walls, then the radial integration will yield zero. In each case, (2) is satisfied. In those cases where the resonant frequencies of the two cavity modes differ, (1b) is not relevant.

Manuscript received April 6, 1993.
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IEEE Log Number 9210811.

[1] E. B. Manring and J. Asmussen, Jr., "Energy and power orthogonality in isotropic, discretely inhomogeneous waveguides," *IEEE Microwave Guided Wave Lett.*, vol. 3, pp. 78–79, Mar. 1993.

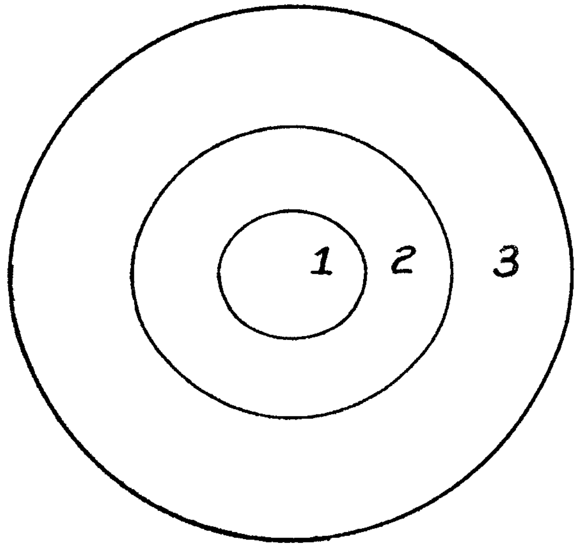

Fig. 1. Cross-section of a circular cylindrical cavity coaxially-loaded with isotropic, discretely inhomogeneous, media.

One can imagine a structure of this type whose inhomogeneity is such that two TE waveguide modes (phase constant k_{zi} and k_{zj}, together with a cavity length L, can be found to give two distinct cavity modes with the same resonant frequency. Then, (1b) applies (based on $k_{zi} \neq k_{zj}$), and (2) applies (since if $k_{zi} \neq k_{zj}$, the axial integration will yield zero). Analogous arguments can be made concerning the orthogonality relations for the magnetic fields of the TM modes.

To summarize, the cavity mode orthogonality relations given by Harrington [1] are completely general (for the media assumptions made) and, in particular, apply to the special cases discussed by the authors.

Authors' Reply[2]

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In Dr. McIssac's comments on our letter,[1] he points out that no contradiction exists between the waveguide orthogonality equations we derived and the cavity orthogonality equations given by [1]. Upon closer inspection, we concur that the apparent contradiction between (15) in our letter[1] and (8-163) of [1] is only apparent.

When performing the cross-sectional integration for two different TE modes, i.e.,

$$\int\int E_i^{\mathrm{TE}} \cdot E_j^{\mathrm{TE}} ds, \tag{1}$$

there arises a factor of $(k_{ci}^2 - k_{cj}^2)$ in the denominator, where the transverse wavenumber k_c is defined by (7) in our letter.[1] In a waveguide where $k_i = k_j$ within each homogeneous region, this quantity is equal to $(k_{zj}^2 - k_{zi}^2)$. Since k_z is the same in every region,

[2]Manuscript received May 11, 1993.
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it is independent of cross-sectional coordinates and may be factored away.

In a lossless cavity, the axial wavenumbers of the two modes must be equal in order that the axial integration be nonzero. When $k_{zi} = k_{zj}$, the quantity $(k_{ci}^2 - k_{cj}^2)$ is given by

$$k_{ci}^2 - k_{cj}^2 = k_i^2 - k_j^2 = \varepsilon\mu(\omega_i^2 - \omega_j^2). \quad (2)$$

The ω's may be factored away, but a factor of $\epsilon\mu$ remains in the denominator. Multiplying the integrand by ϵ for the cavity case then has the same effect as dividing the integrand by μ for the waveguide case.

We would like to express our appreciation to Dr. McIssac for clarifying this point.

References

[1] R. F. Harrington, *Time-Harmonic Electromagnetic Fields.* New York: McGraw-Hill, 1961.